A State-space model estimation of EEG signals using subspace identification

2102499 : Electrical Engineering Senior Project Presentation

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OUTLINE

- Introduction
- Methodology
- Result & Discussion
- Conclusion

The measure bring relevant information about the activity from activated network

Focuses on identifying EEG sources in state space model and AR model

Learn brain connectivity for EEG signal by using Granger causality test on state space model and AR model.

METHODOLOGY

Estimate EEG model described by state space

Estimate EEG model described by AR model

Subspace Method

- Divided data into two parts denote as past and future data
- Project future data on past data
- Solve least square for system matrices

AR model

$$y(t) = \sum_{i=1}^{p} A_i y(t-i) + v(t)$$

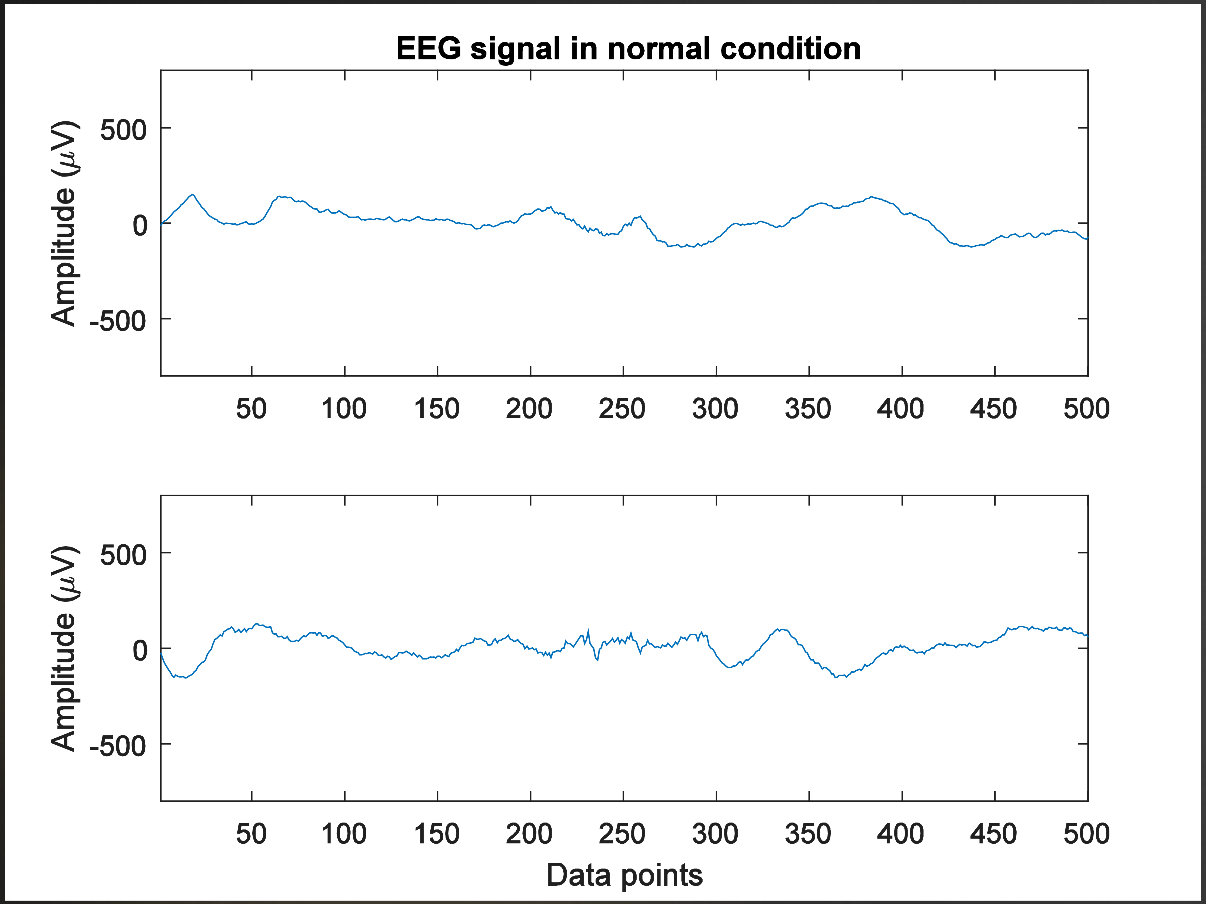
State-space model

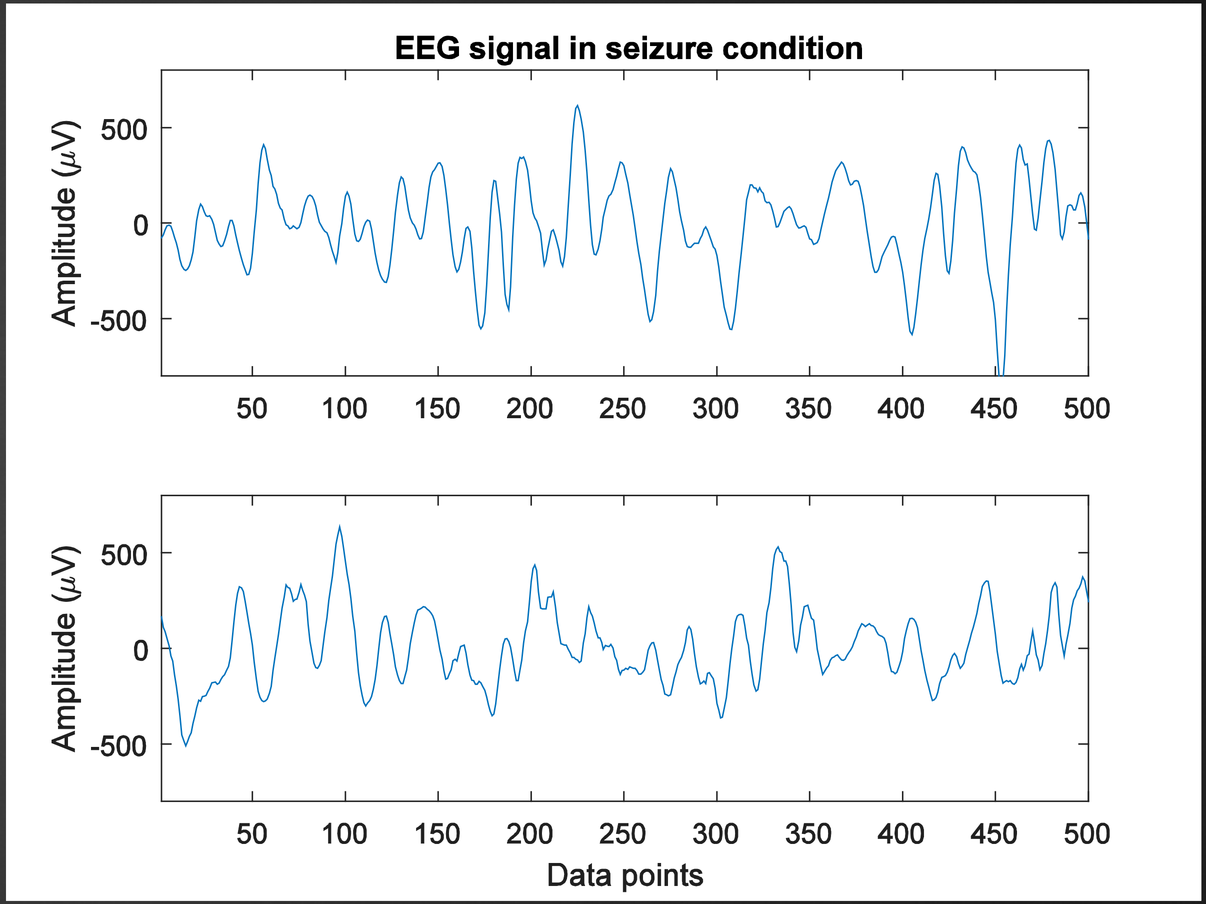
$$x(t+1) = Ax(t) + Kv(t)$$
$$y(t) = Cx(t) + v(t)$$

Maximum likelihood

- Use maximum likelihood to estimate parameters AR(p)
- Choose model order by using Bayesian Information Criteria (BIC) scores.

Example of EEG data for two channels





Classification of EEG data

Differences in frequencies

Pole Location

- Oscillation characteristic of two data types.
- Hypothesis : EEG data in two conditions have different rate of oscillation.

$$\theta = tan^{-1}\left(\frac{Im(\lambda)}{Re(\lambda)}\right)$$

λ is eigenvalue of system

Differences in amplitude

$\|H\|_2$ -norm (Average energy)

- Steady state power of output response.
- Hypothesis : EEG data in two conditions have different level of energy, in average.

$$\|G\|_2 = \left(\frac{1}{2\pi}\int_{-\pi}^{\pi} Trace\left[G\left(e^{j\omega}\right)^H G\left(e^{j\omega}\right)\right]\, d\omega\right)$$

$\|H\|_\infty$ -norm (Peak gain)

- Frequency at peak gain or largest singular value occurs.
- Hypothesis : EEG data in two conditions have different peak gain.

$$\|G\|_\infty = sup_{\omega\in(-\pi,\pi)}\left|G\left(e^{j\omega}\right)\right|$$

METHODOLOGY

Learning brain connectivity

State-space model

$$x(t+1) = Ax(t) + w(t)$$
$$y(t) = Cx(t) + v(t)$$

AR model

$$y(t) = \sum_{i=1}^{p} A_i y(t-i) + v(t)$$

Granger Causality test

- **State-space model**: Reduce model by removing j^{th} row of C
- Solve prediction error from Riccati equation

$$\Sigma = A\Sigma A^T + W - A\Sigma C^T (C\Sigma C^T)^{-1} C\Sigma A^T$$

- Determine time-domain Granger causality (Seth,2015)

$$F_{y_j \to y_i | All\ others\ y} = \log \frac{|\Sigma_{ii}^R|}{|\Sigma_{ii}|}$$

If $|\Sigma_{ii}^R| = |\Sigma_{ii}|$, y_j does not cause y_i

- **AR model**:
If $(A_k)_{ij} = 0 \;\; ; \quad \forall k\,,\ y_j$ does not cause y_i

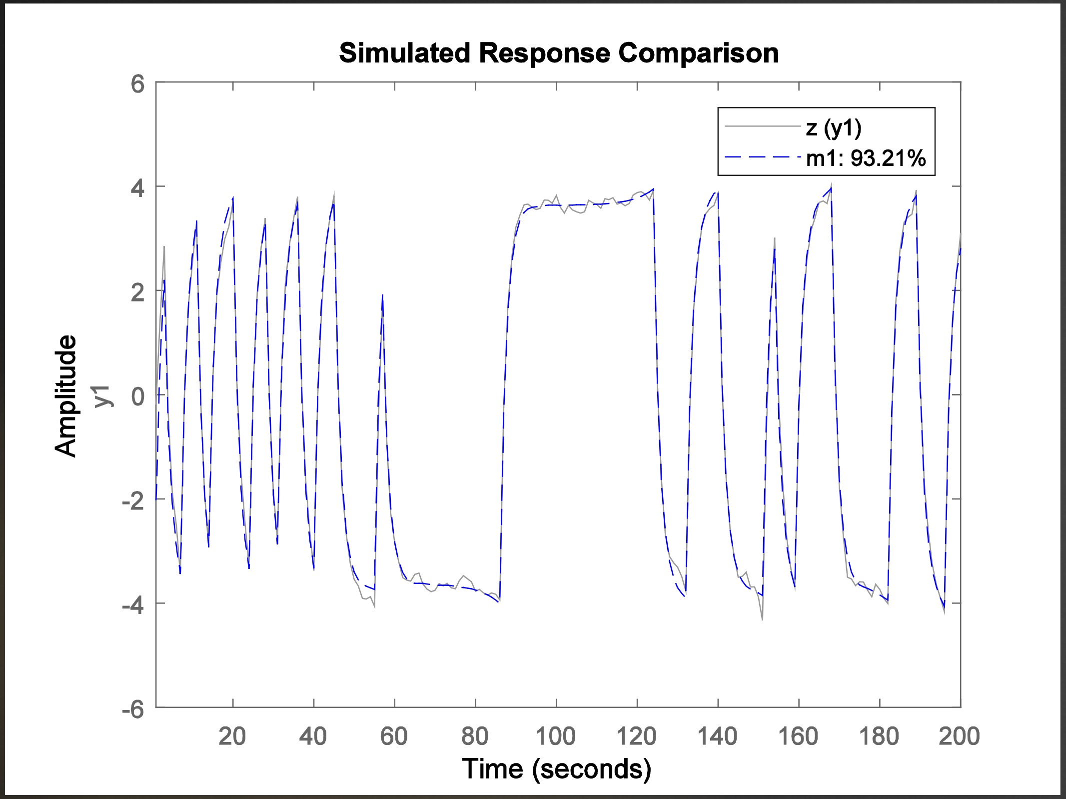
Experimental Results

Model estimation on EEG signals

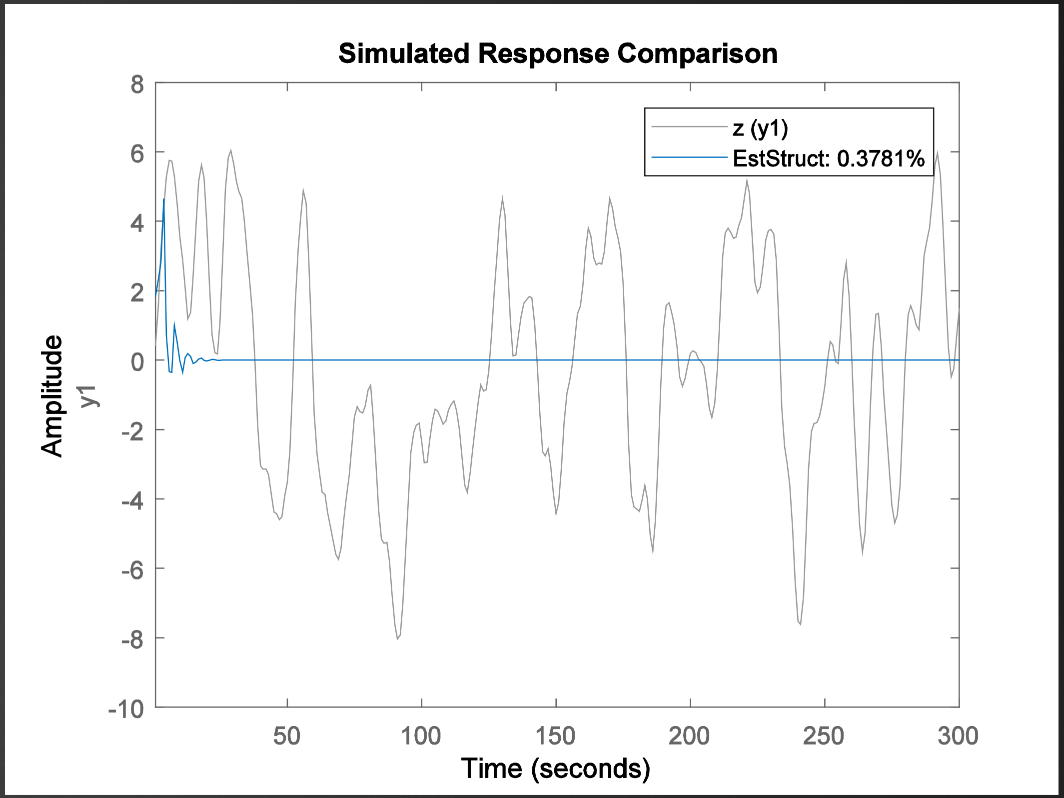
Classification on EEG data

GC test on normal EEG data

State-space estimation on simulated data

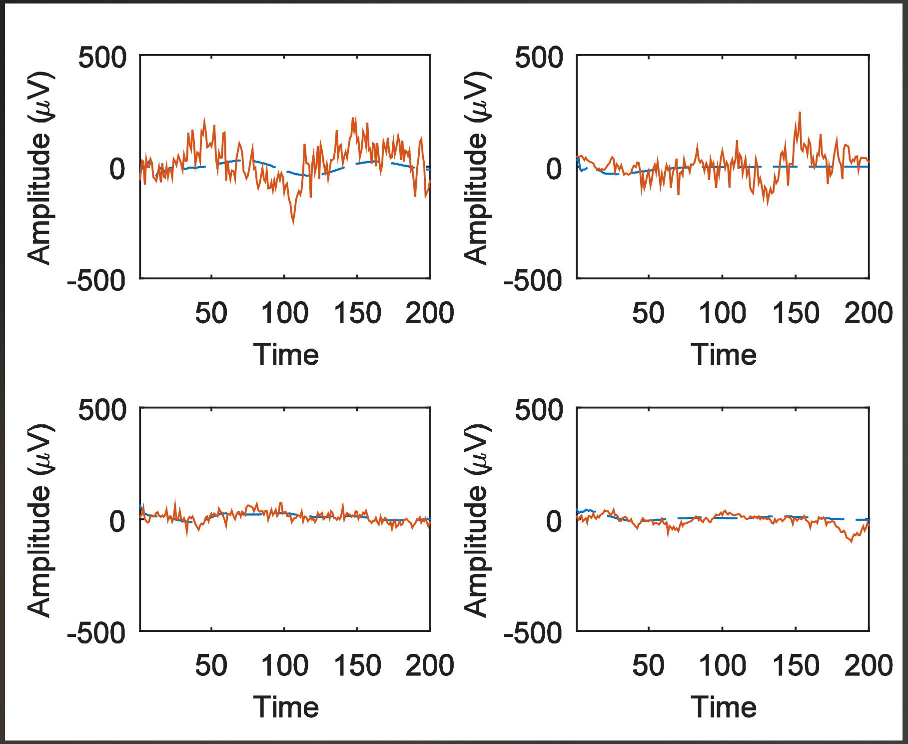


Ground-truth state space model with deterministic input

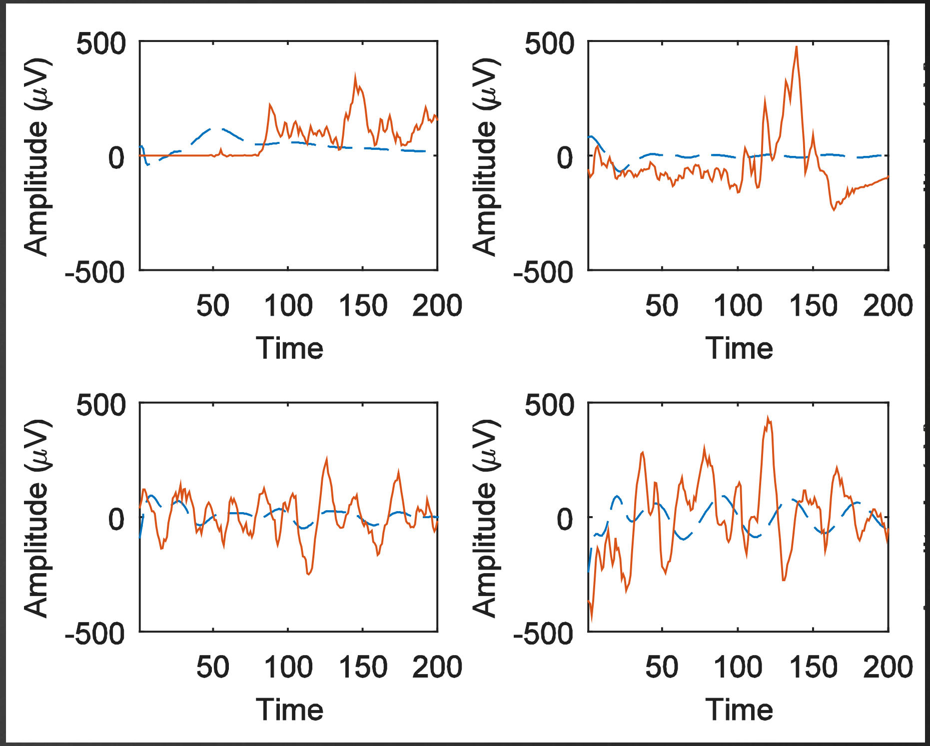


Ground-truth state space model without deterministic input

Experimental Results
State-space estimation on EEG data



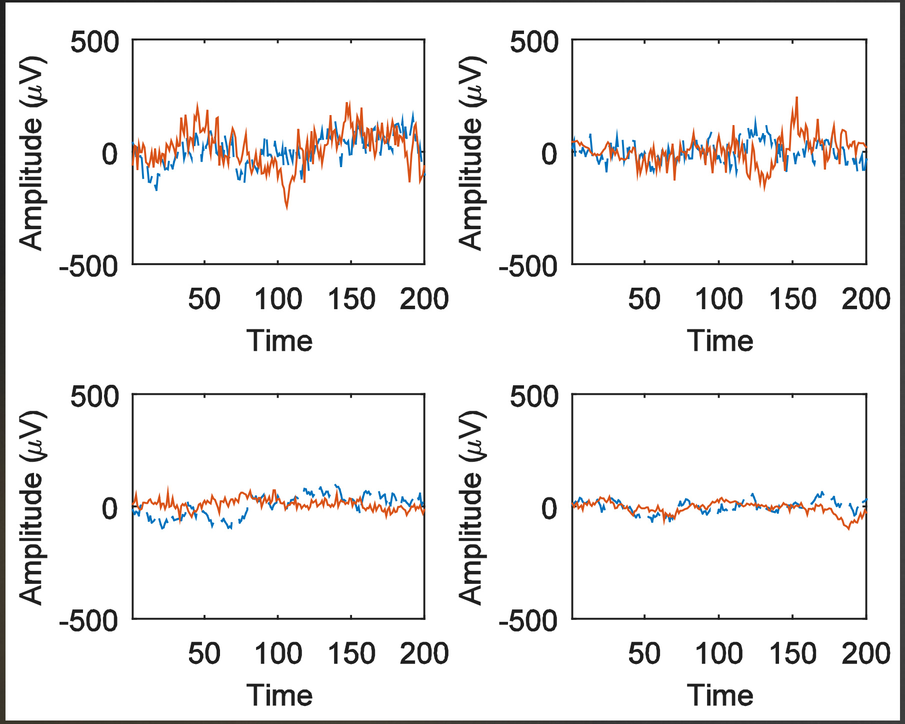
Normal data



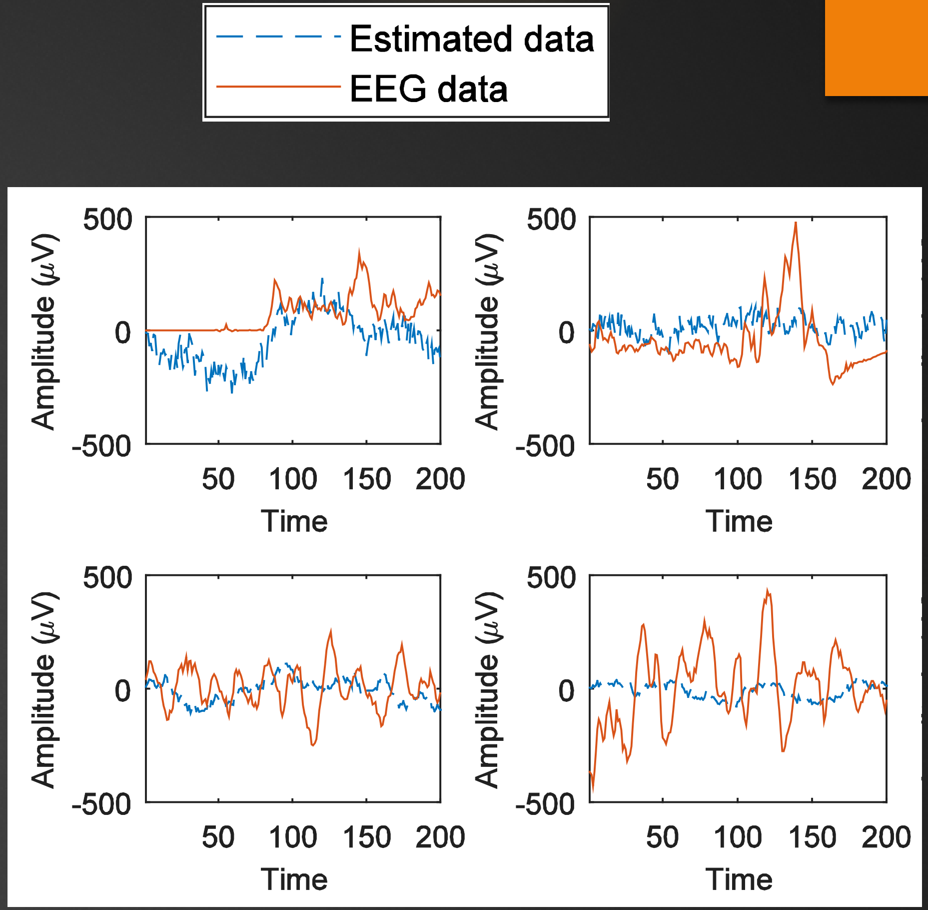
Seizure data

State-space models are with order 25 and stable

Experimental Results
AR model estimation on EEG data



Normal data



Seizure data

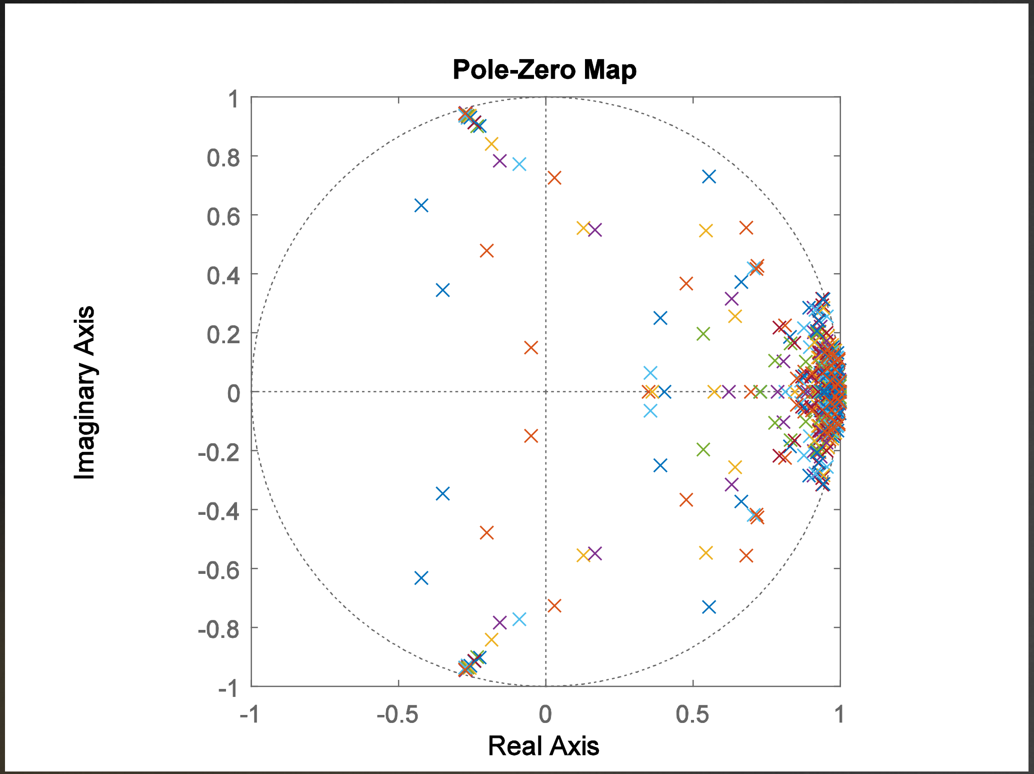
Experimental Results
Mean square error of estimation

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\left(Y_i - \widehat{Y}_i\right)^2$$

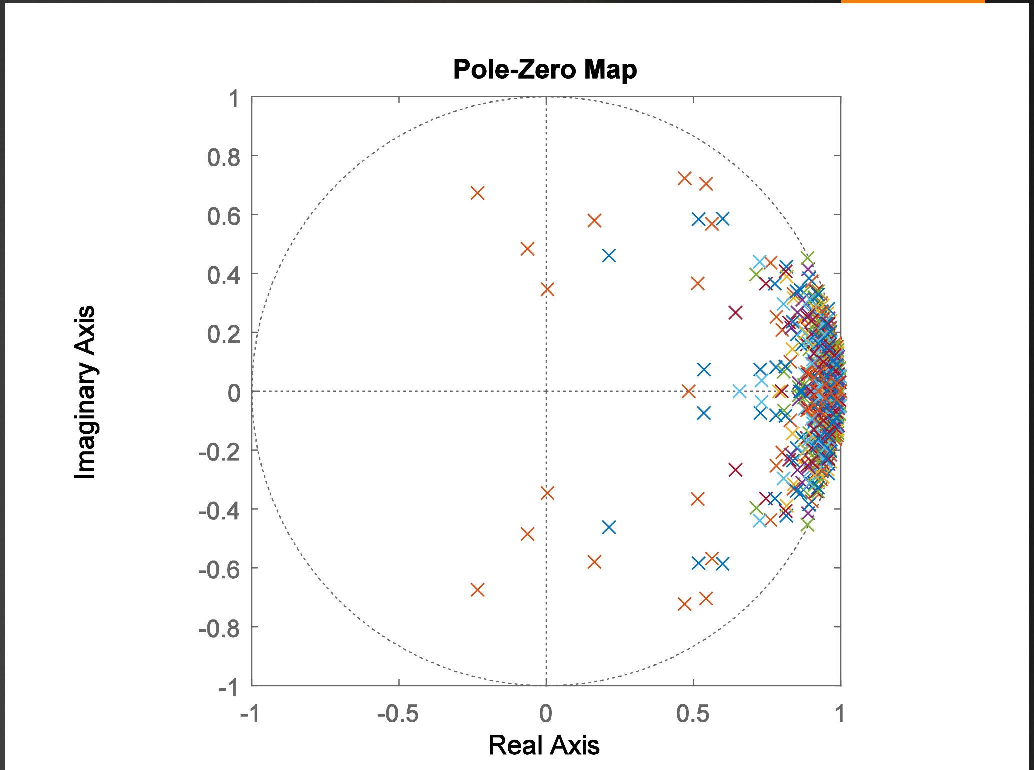
Mean square error of estimation for each trial in average

	Normal Data	Seizure Data
State-space model	1.5370×10^4	1.7191×10^5
AR model	4.9625×10^4	2.9133×10^5

Pole location of EEG Data



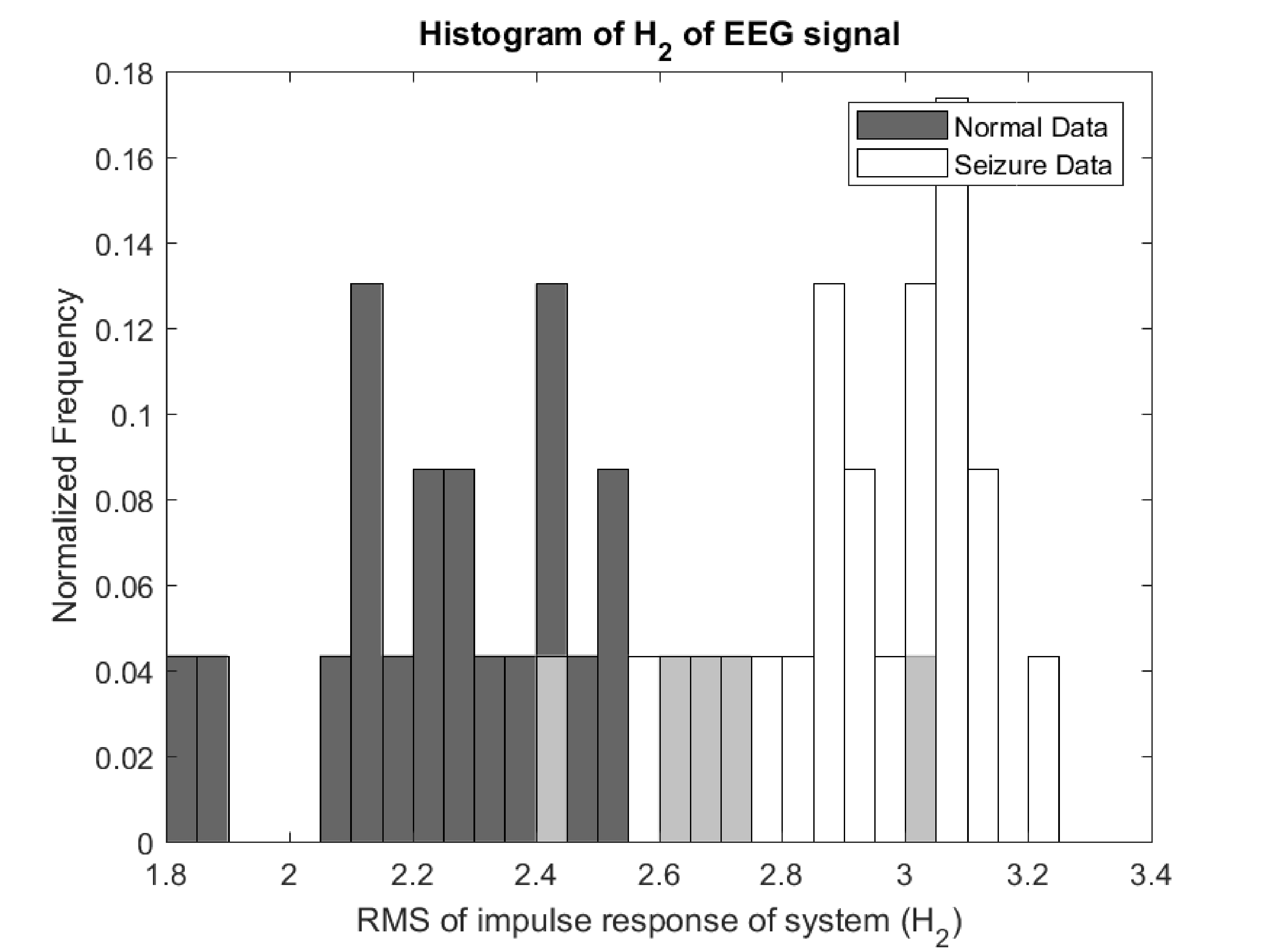
Normal Data



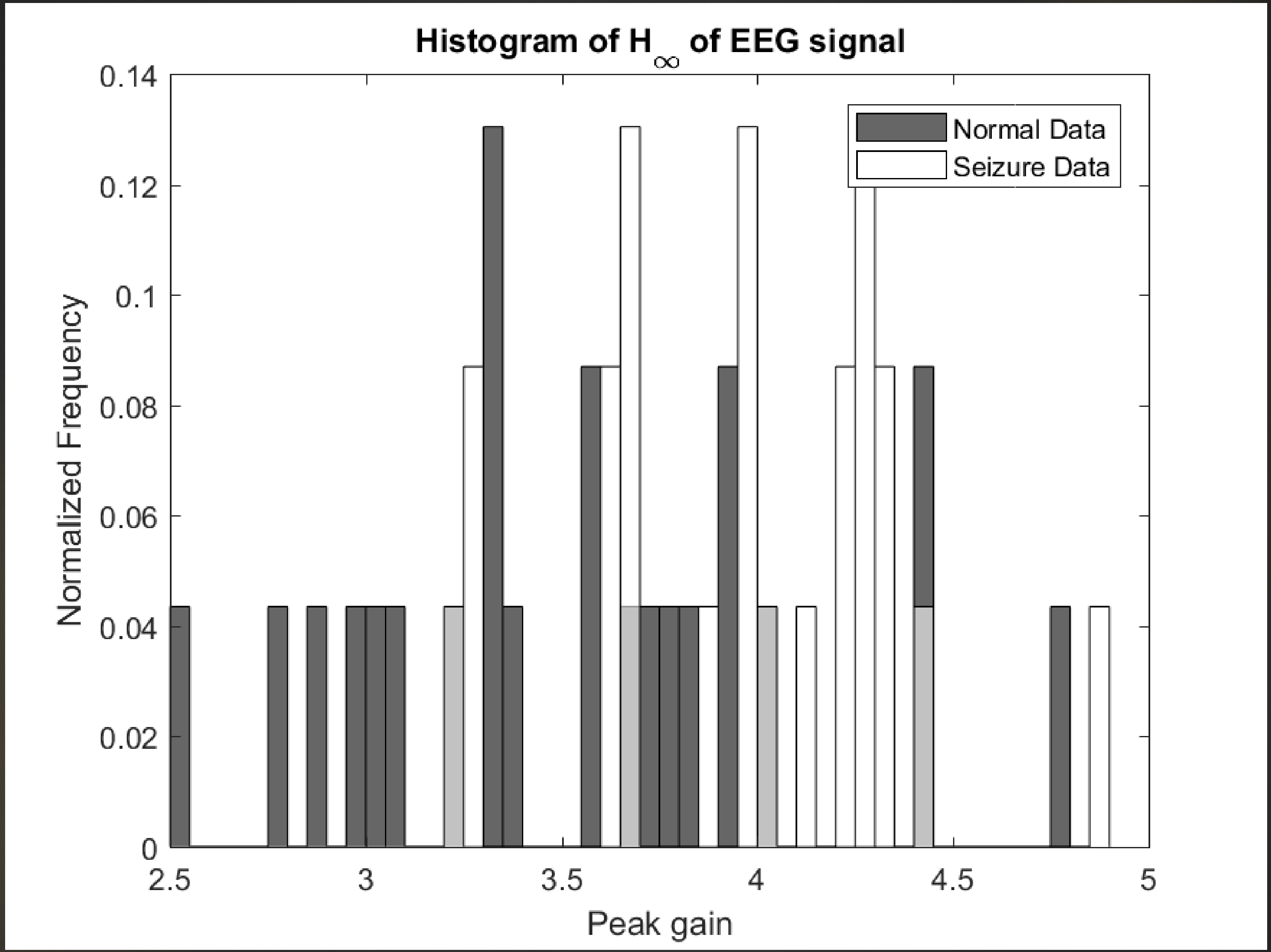
Seizure Data

We observed the angle of pole which tell us about system frequencies.

Classification of EEG data

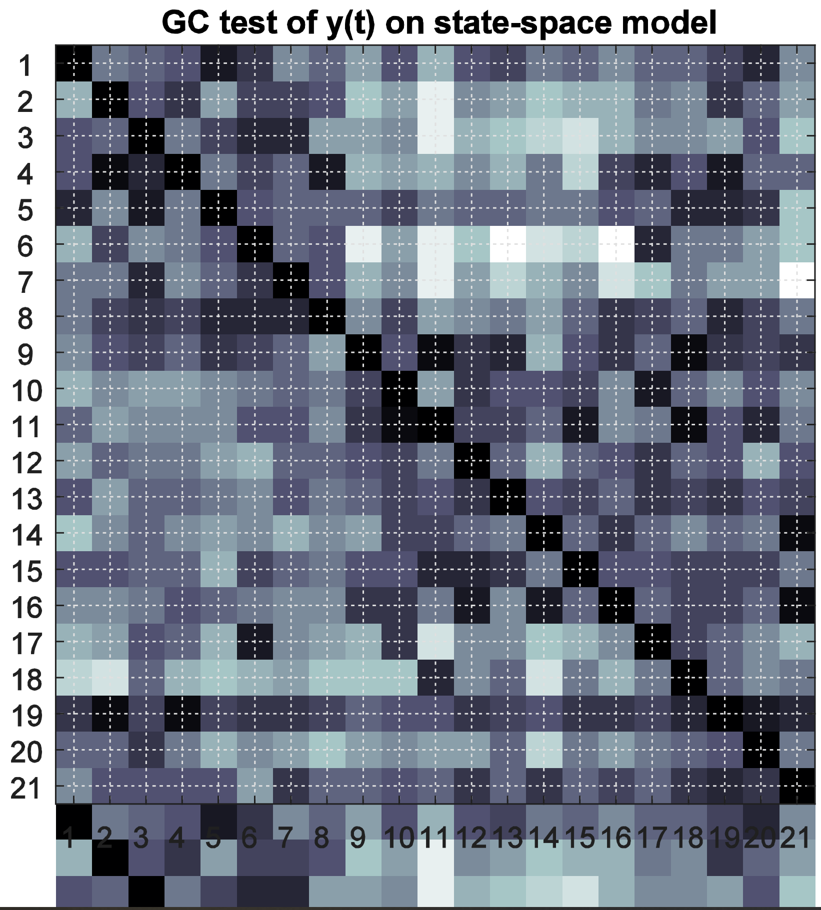


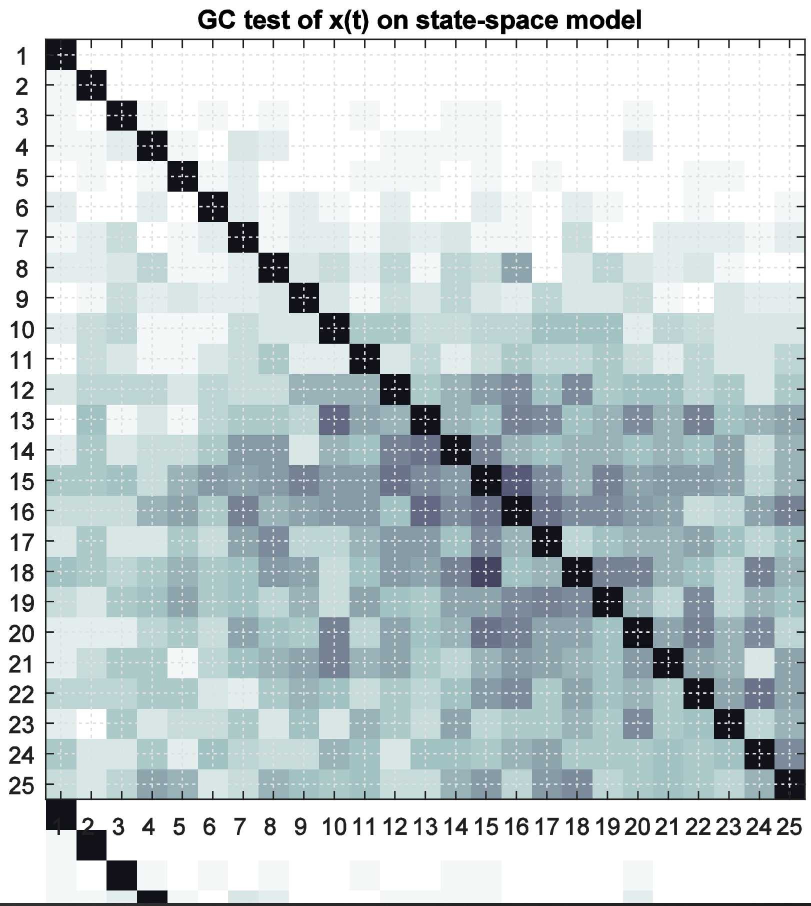
Histogram of H2 of EEG signal
Normalized Frequency
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1.8
2
2.2
2.4
2.6
2.8
3
3.2
3.4
RMS of impulse response of system (H2)
Normal Data
Seizure Data


Histogram of H_∞ of EEG signal
Normal Data
Seizure Data
Normalized Frequency
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
2.5
3
3.5
4
4.5
5
Peak gain

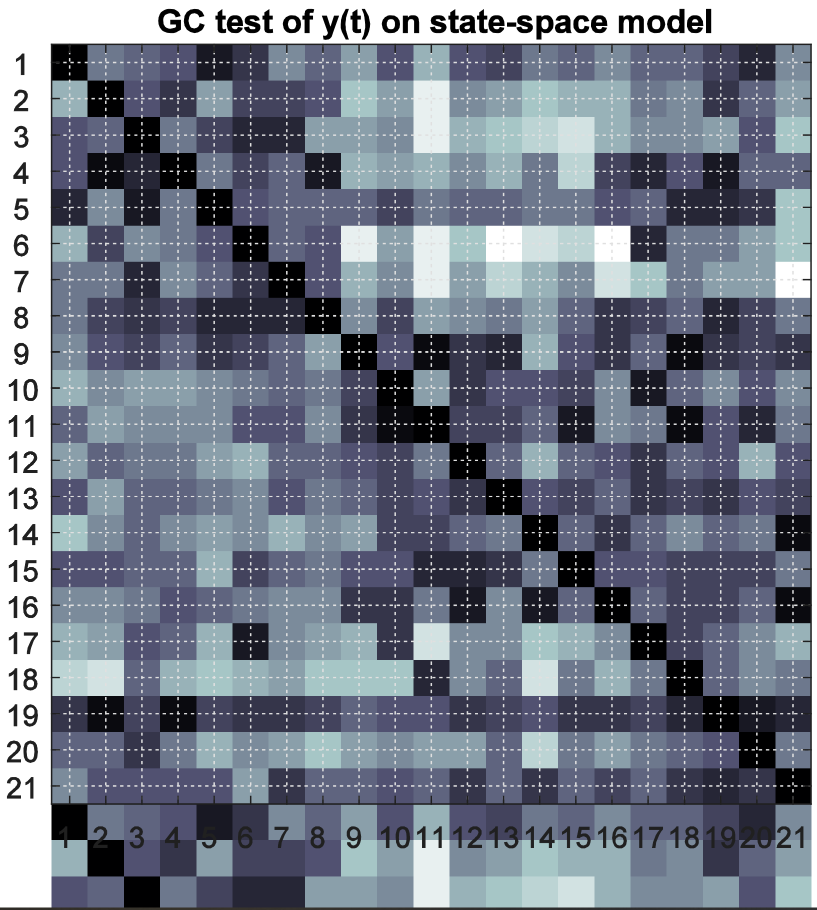
Experimental Results
Brain connectivity on state-space model

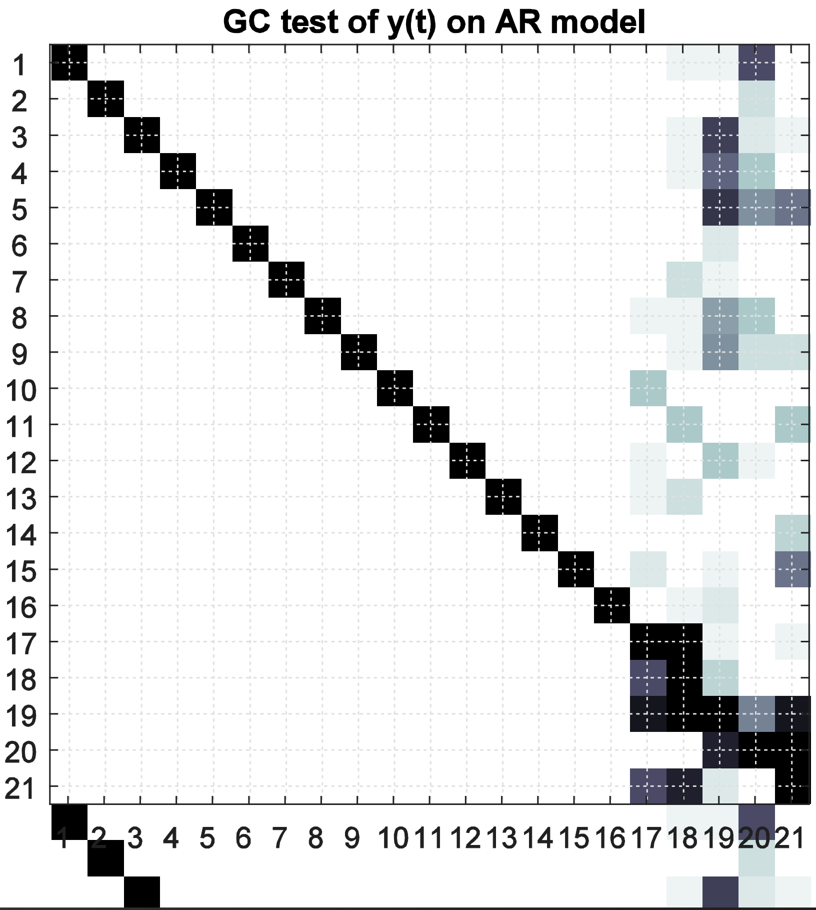
x(t) : EEG Sources
y(t) : EEG signals





Experimental Results
Brain connectivity of y(t)





Conclusion

- Both state-space model and AR model have poor fitting. State-space model tends to be more accurate than AR model
- H_2-norm is only feature that can classify EEG data but we cannot make sure that the others feature are not work because we use model from previous experiment
- Brain connectivity of source is sparser than EEG signal brain connectivity
- If we use statistical test on state space model brain connectivity, the result may tell causality relation difference between state-space model and AR model

Q&A

References

- L. Barnett and A. K. Seth, "Granger causality for state space models," *Physical Review E*, vol. 91, no. 4, 2015.
- A. Pruttiakaravanich and J. Songsiri, "A review on dependence measures in exploring brain networks from fMRI data," *Engineering Journal*, vol. 20, no. 3, pp. 207-233, 2016
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