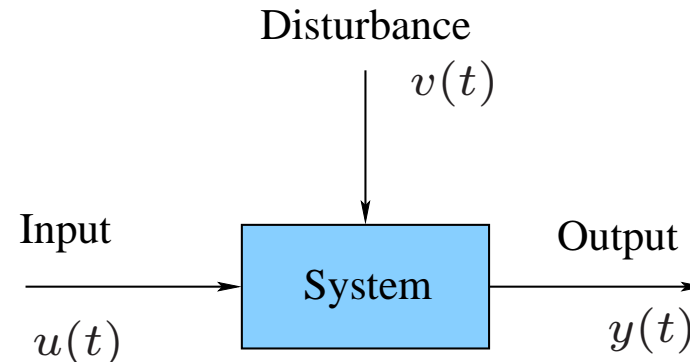


1. Introduction

- basic concept
- system identification methods
- procedures in system identification
- examples

Basic concept

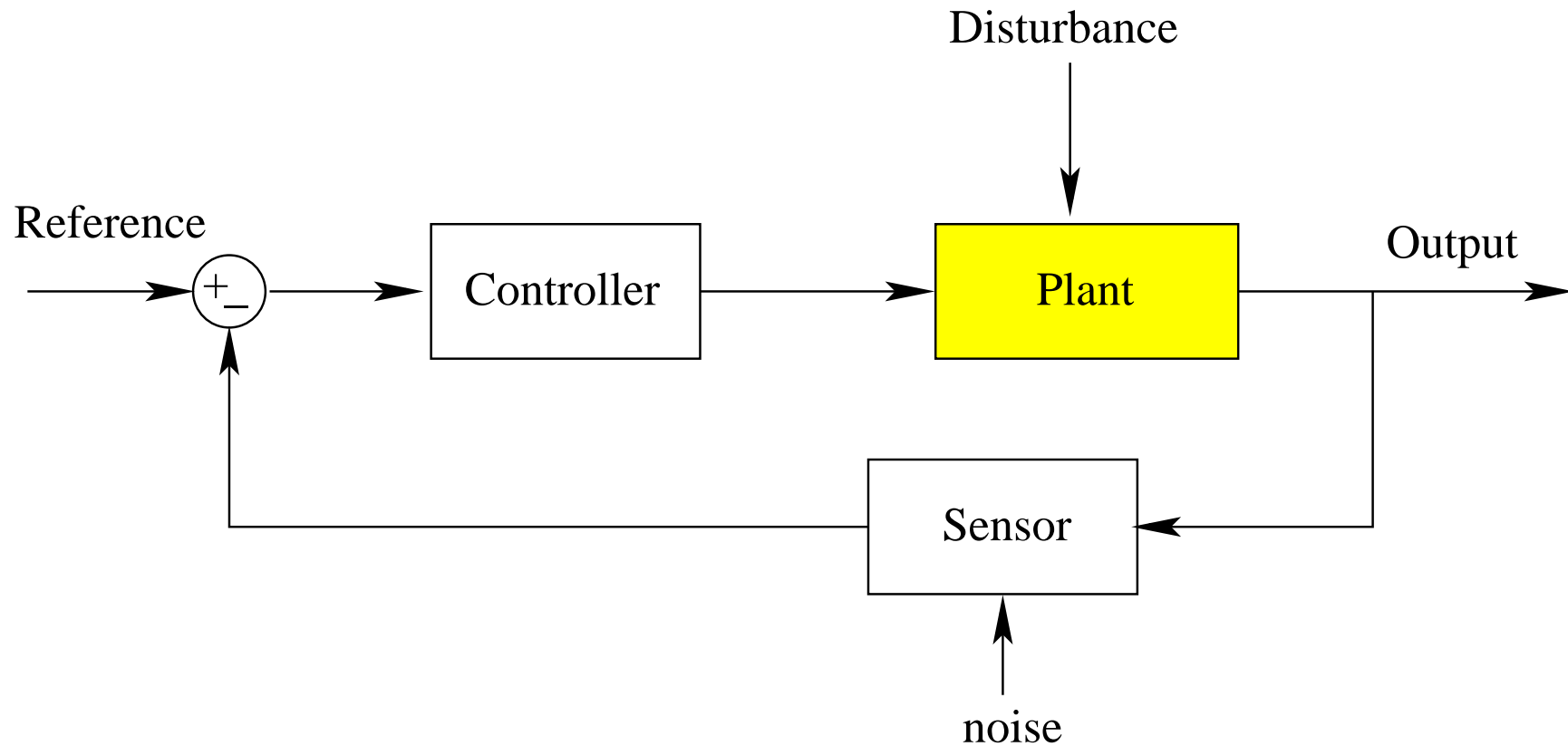
objective: how to build a system description from experimental data



estimation of system description can serve for many purposes:

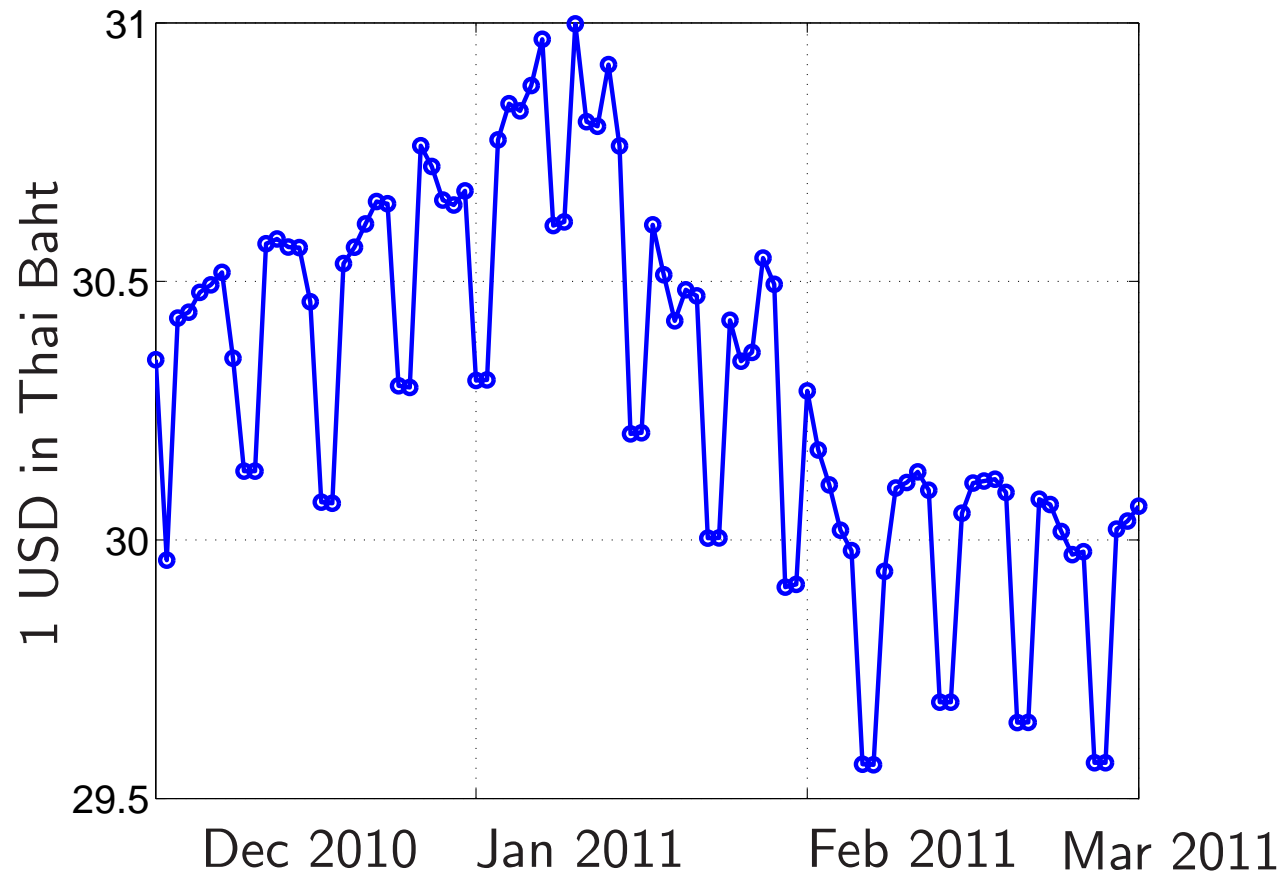
- obtain a mathematical model for controller design
- explain/understand observed phenomena (e.g., machine learning)
- forecast events in the future (e.g., time series analysis in econometrics)
- obtain a model of signal in filter design (e.g., signal processing)

System Identification for Controller Design



- for controller design, the plant is assumed known
- in system identification, we aim to estimate the parameters in a model

System Identification for prediction



how to forecast the Thai Baht in Apr, May,... ?

need a **model** for prediction, e.g. $\hat{x}_{\text{Apr}} = a_1 x_{\text{Mar}} + a_2 x_{\text{Feb}}$

Models

a description of the system, or a relationship among observed signals

a model should capture the essential information about the system

Types of Models

- graph and tables, e.g., bode plots and step response
- mathematical models, e.g., differential and difference equations
- probabilistic models, e.g., probability density function

System identification is a process of obtaining models based on a data set collected from experiments

input and output signals from the system are recorded and analyzed to infer a model

System identification methods

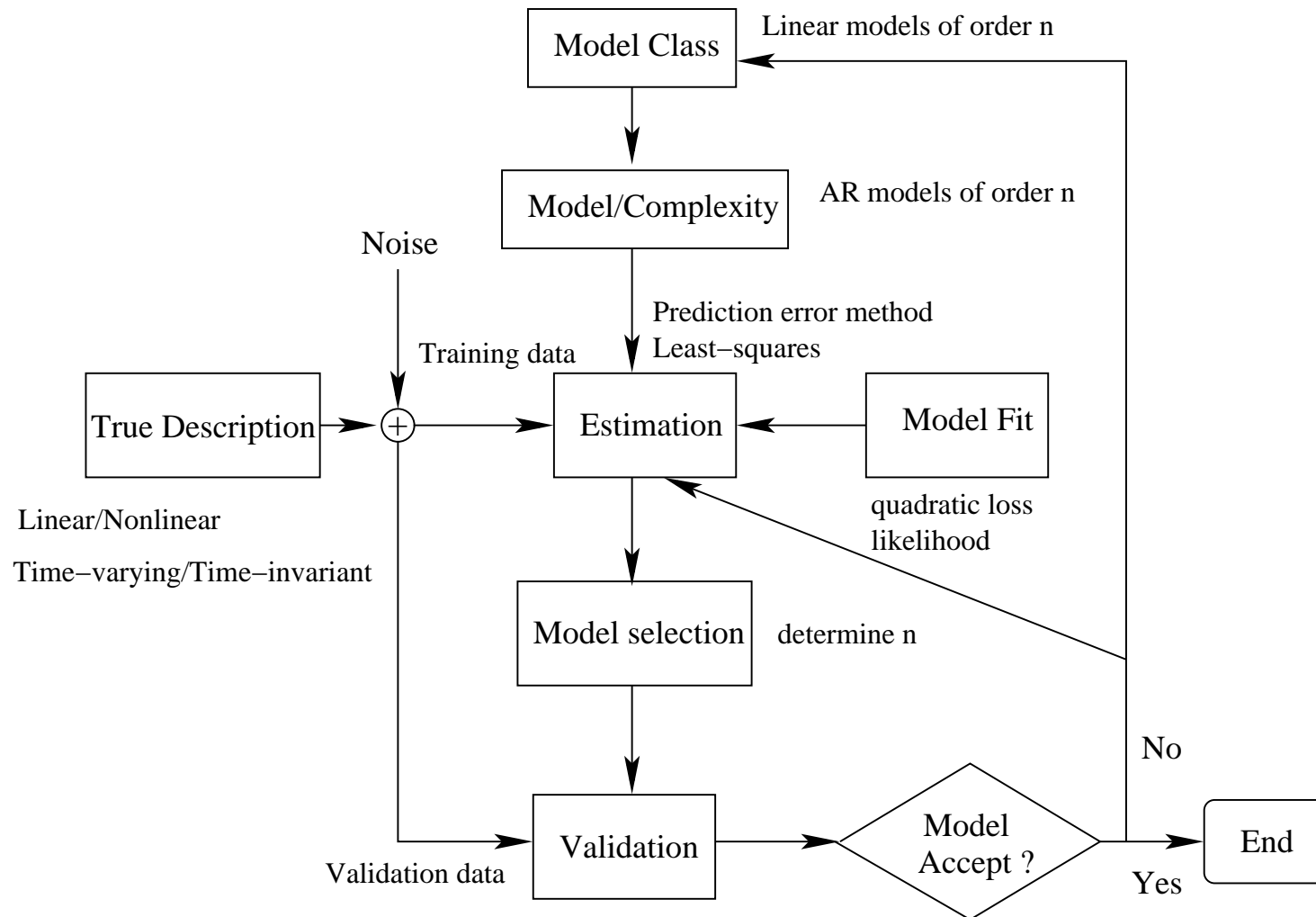
- **Nonparametric approach**

- aim at determining a (time/frequency) response directly without first selecting a possible set of models
- gives basic information about the system and is useful for validation
- examples are transient analysis, frequency analysis, correlation analysis, and spectral analysis

- **Parametric approach**

- require assumptions on a model class/structure
- the search for the best model within the candidate set becomes a problem of determining the model parameters
- typically more complicated than the nonparametric approach
- results can be further used for controller design, simulation, etc.

Procedures in System Identification



Parametric Estimation

- Model classification:

SISO/MIMO, Linear/Nonlinear, Time-invariant/Time varying,
Discrete/Continuous

- searching the best model within a candidate set becomes a problem of determining the model parameters
- the selected parameter \hat{x} from a model class \mathcal{M} is optimal in some sense, i.e.,

$$\hat{x} = \operatorname{argmin}_{x \in \mathcal{M}} f(x, \mathcal{D}),$$

where f is a measure of goodness of fit (or loss function) and is a function of information data (\mathcal{D})

- examples of f are quadratic loss, likelihood, entropy function, etc.

Estimation methods

- linear least-squares method (LS)
simple to compute, no assumption on noise model
- statistical estimation methods, e.g., Maximum likelihood, Bayes
use prior knowledge about the noise
- instrumental-variable method
a modification of the LS method for correlated noise
- prediction-error method
model the noise, applicable to a broad range of models

Model selection

- **Principle of parsimony:**

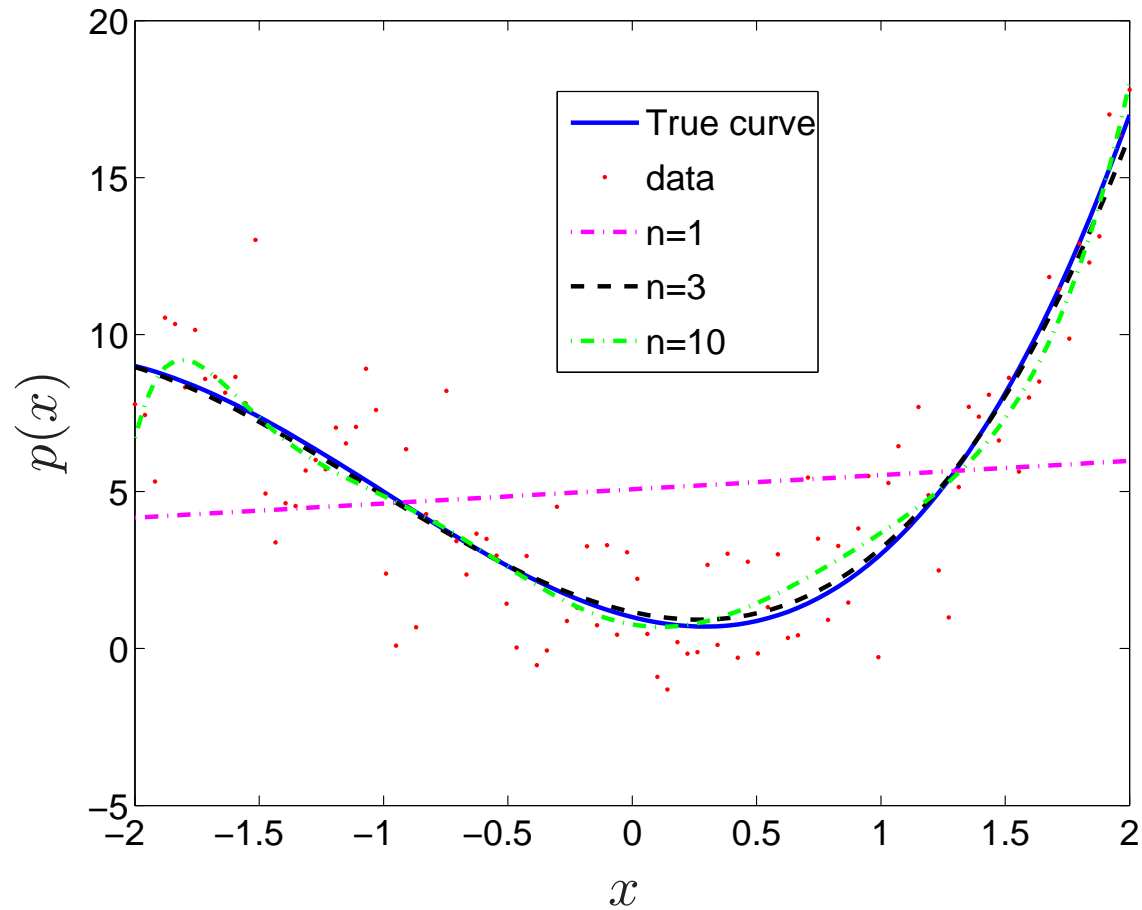
one should pick a model with the smallest possible number of parameters that can adequately explain the data

- one can trade off between

Goodness of fit VS Complexity

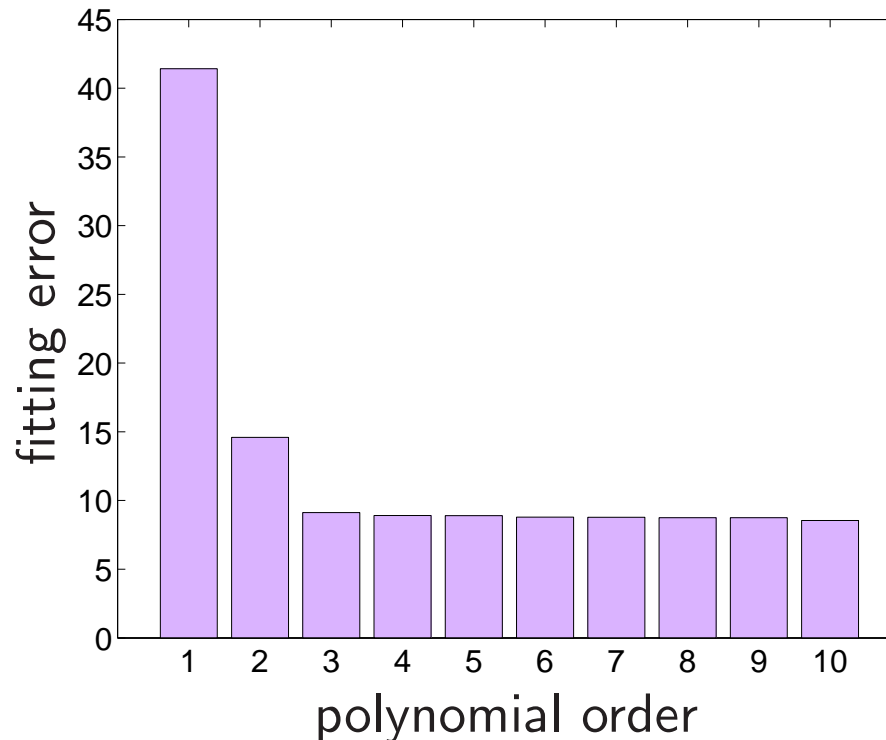
- related to the concept of bias VS variance in statistics
- examples of model selection criteria are FPE, AIC, BIC, etc.

Example: Polynomial fitting



- a curve fitting problem of polynomial of order n
- the true order is $n = 3$

Example: Trade-off curve

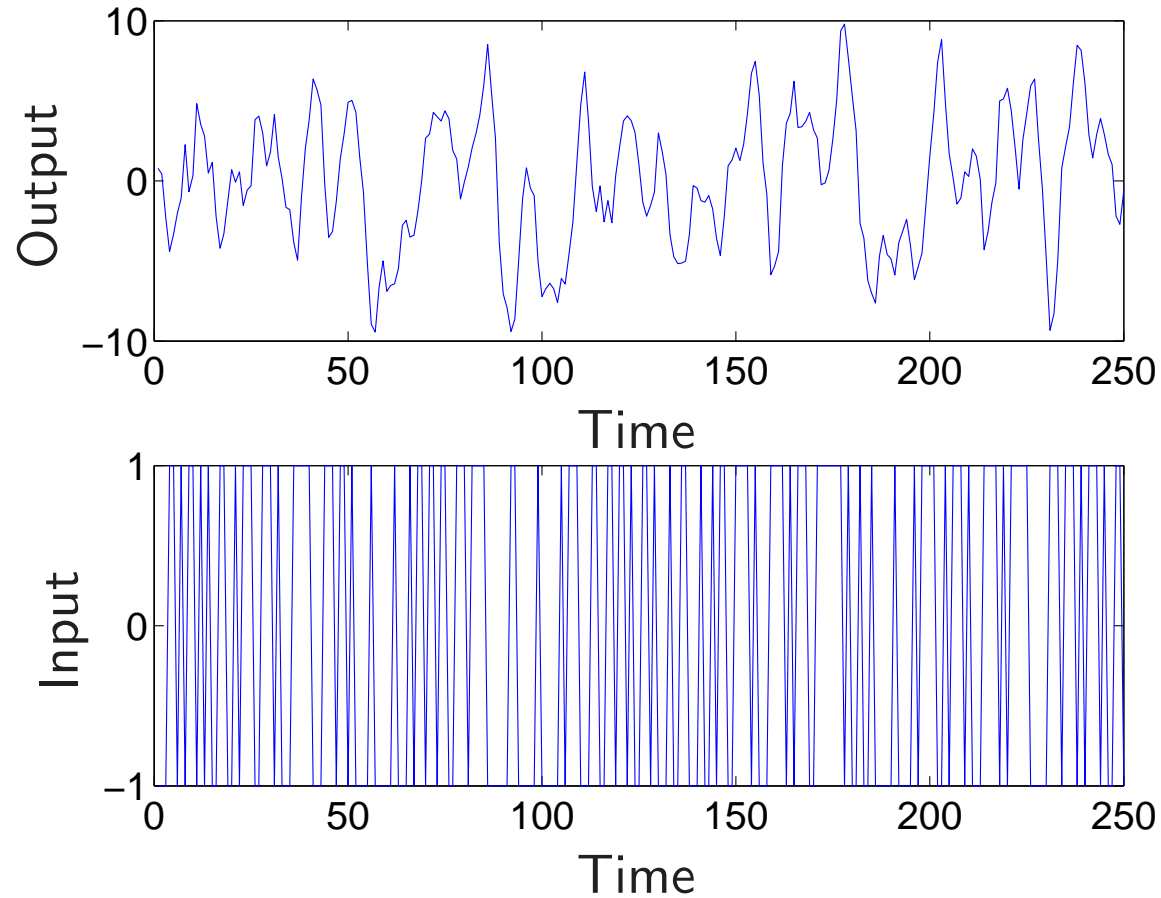


- shows the minimized loss as a decreasing function of model complexity
- the error begins to decrease as the model picks up the relevant features
- as the model order increases, the model tends to *over fit* the data
- In practice, the model order is determined by the “knee” in the curve

Model Validation

- a parametric estimation procedure picks out the *best* model
- a problem of model validation is to verify whether this best model is “good enough”
- test the estimated model (obtained from training data), with a new set of data (validation set)
- The tests verify whether the dynamic from the input and the noise model are adequate

Numerical Example



- feed a known input to the system and measure the output
- the input should contain rich information to excite the system

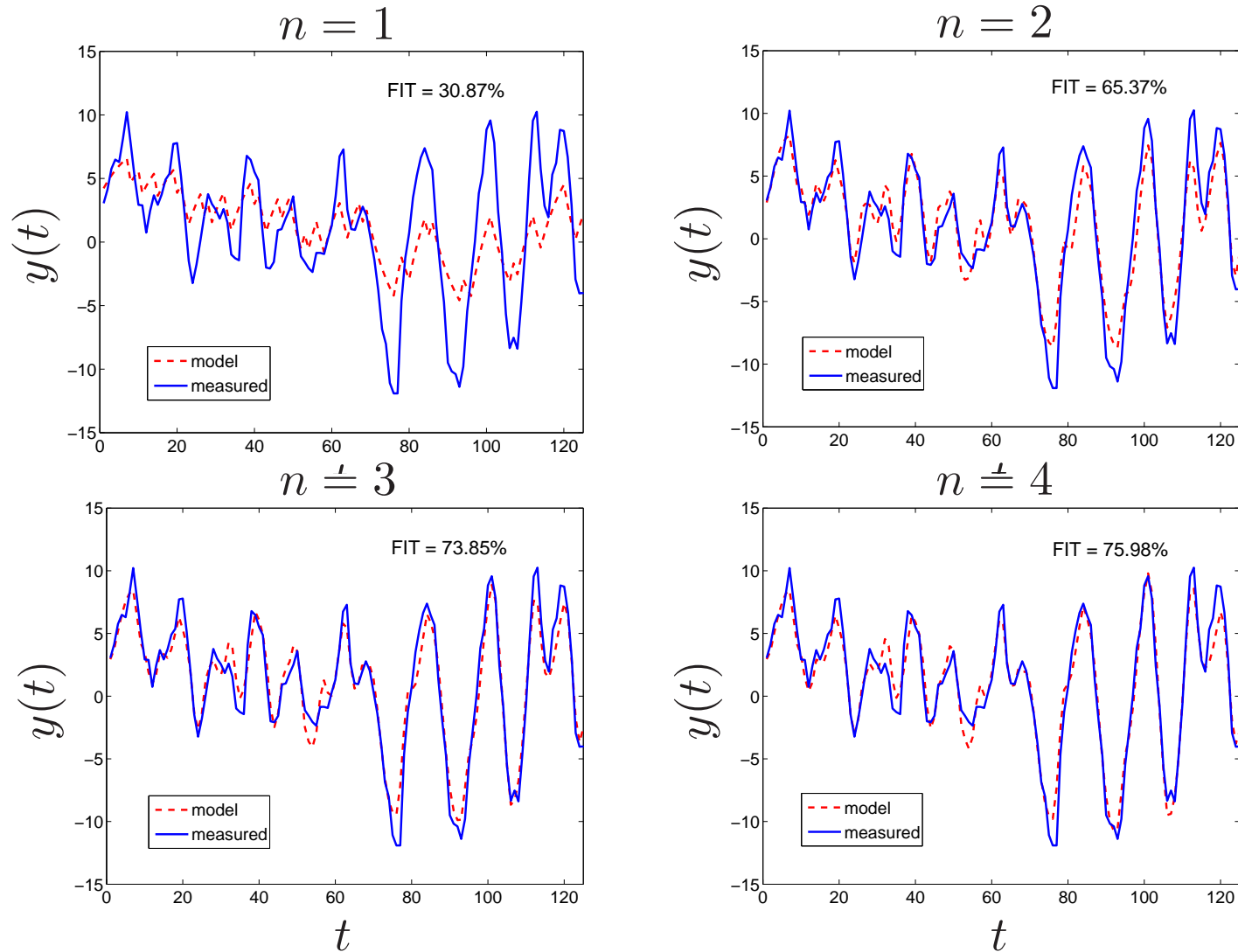
- fit the measured output to the model

$$(1 + a_1q^{-1} + \dots + a_nq^{-n})y(t) = (b_1q^{-1} + \dots + b_nq^{-n})u(t) + (1 + c_1q^{-1} + \dots + c_nq^{-n})\nu(t)$$

with unknown parameters $a_1, \dots, a_n, b_1, \dots, b_n, c_1, \dots, c_n$

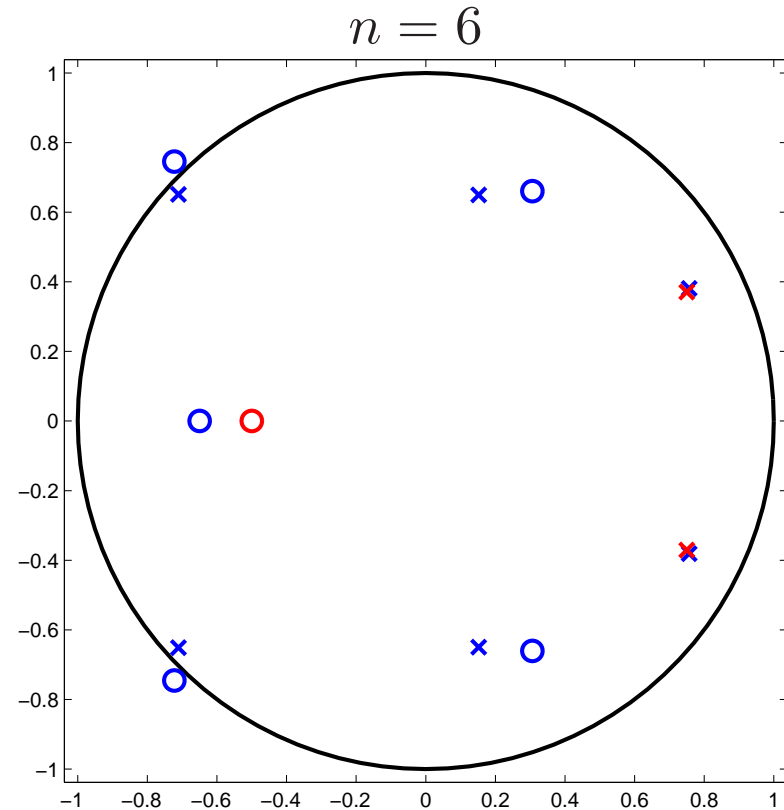
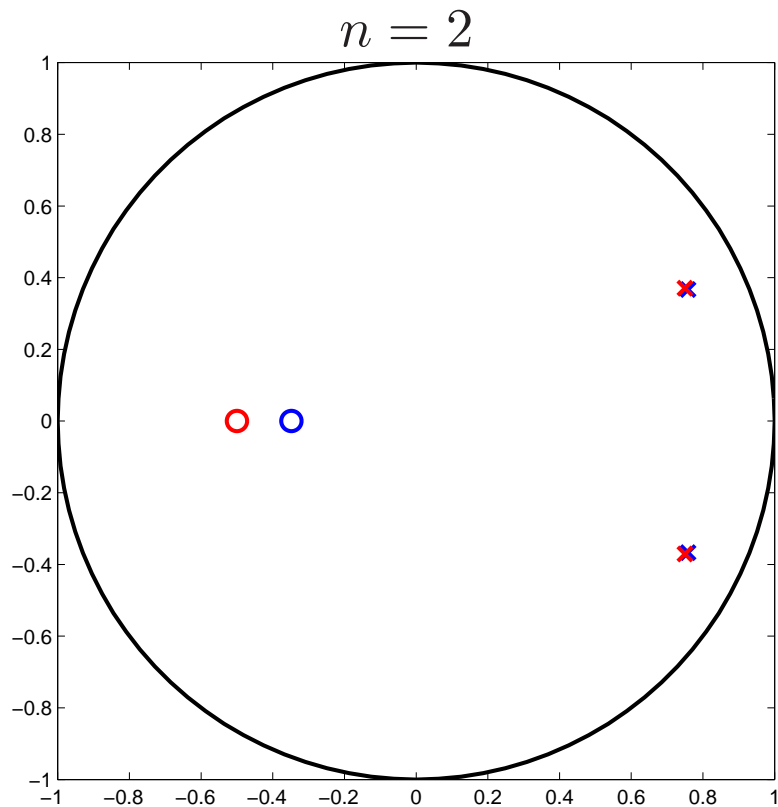
- this model is known as *Autoregressive Moving Average with Exogenous input (ARMAX)*
- $\nu(t)$ represents the noise that enters to the system
- n is the model order, which is selected via *model selection*
- the parameters are estimated by the *Prediction-error method (PEM)*

Example of output prediction



(estimated by PEM and validated on a new data set)

Example of Zero-Pole location



- ○: zeros, ×: poles
- red: true system, blue: estimated models
- chance of zero-pole cancellation at higher order

Skills needed for System Identification

one should have

- concepts of dynamical systems (description, how to analyze their properties)
- probability and statistics (to understand probabilistic models, estimation methods, to statistically interpret results)
- linear algebra (many linear models involve matrix analysis)
- optimization (most model estimations are optimization problems)
- programming (for numerical methods to solve estimation problems)

References

Chapter 1,2 in

L. Ljung, *System Identification: Theory for the User*, Prentice Hall, Second edition, 1999

Chapter 1-3 in

T. Söderström and P. Stoica, *System Identification*, Prentice Hall, 1989

L. Ljung, *Perspective on System Identification*,
<http://www.control.isy.liu.se/~ljung/>